

**WIUT Math Test**  
**8 April 2009**

**Question 1**

Solve  $x^3 - x^2 - 20x = 0$ .

**Solution:**

$$x(x^2 - x - 20) = 0$$

1)  $x = 0$ ;

2)  $x^2 - x - 20 = 0$

$$D = (-1)^2 - 4 \cdot 1 \cdot (-20) = 1 + 80 = 81$$

$$x_1 = \frac{-(-1) + \sqrt{81}}{2 \cdot 1} = \frac{1 + 9}{2} = 5; \quad x_2 = \frac{-(-1) - \sqrt{81}}{2 \cdot 1} = \frac{1 - 9}{2} = -4.$$

Answer:  $x = -4$ ;  $x = 0$  or  $x = 5$ .

**Question 2**

Simplify  $A = \frac{a^2 - a - 20}{25a - a^3} \times \frac{a^2 - a - 2}{(a-2)(a+4)} \times \frac{a+1}{a^2 + 5a}$

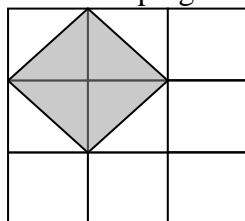
**Solution:**

$$\begin{aligned} A &= \frac{a^2 - a - 20}{25a - a^3} \times \frac{a^2 - a - 2}{(a-2)(a+4)} \times \frac{a+1}{a^2 + 5a} = \frac{a^2 - a - 20}{25a - a^3} \times \frac{a^2 - a - 2}{(a-2)(a+4)} \cdot \frac{a+1}{a^2 + 5a} = \frac{a^2 - a - 20}{25a - a^3} \times \frac{a^2 - a - 2}{(a-2)(a+4)} \times \frac{a^2 + 5a}{a+1} = \\ &= \frac{(a-5)(a+4)}{a(5-a)(5+a)} \times \frac{(a-2)(a+1)}{(a-2)(a+4)} \times \frac{a(a+5)}{a+1} = \frac{a-5}{5-a} = \frac{-(5-a)}{5-a} = -1. \end{aligned}$$

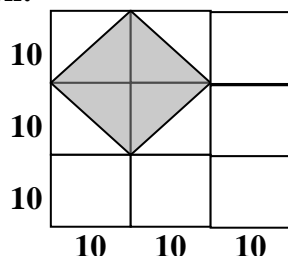
Answer:  $A = -1$ .

**Question 3**

A 30x30 meter garden is split into 9 equal squares as shown. The shaded areas are planted with flowers, at a cost of 10,000 Uzbek soom per square meter. The white areas are cleaned up every month at a cost of 1,000 Uzbek soom per square meter. If planting happens only once a year, and cleaning every month, what is the yearly cost of keeping the garden?



**Solution:**



Yearly cost = Planting cost + Cleaning cost

Planting cost = Shaded Area x Unit cost =  $\frac{1}{2} \cdot 20 \cdot 20 \times 10,000 = 200 \times 10,000 = 2,000,000$ .

Cleaning cost = 12 x White area x Unit cost = 12 x (Full area - Shaded area) x 1,000 = 12 x (30x30 - 200) x 1,000 = 12 x 700 x 1,000 = 8,400,000.

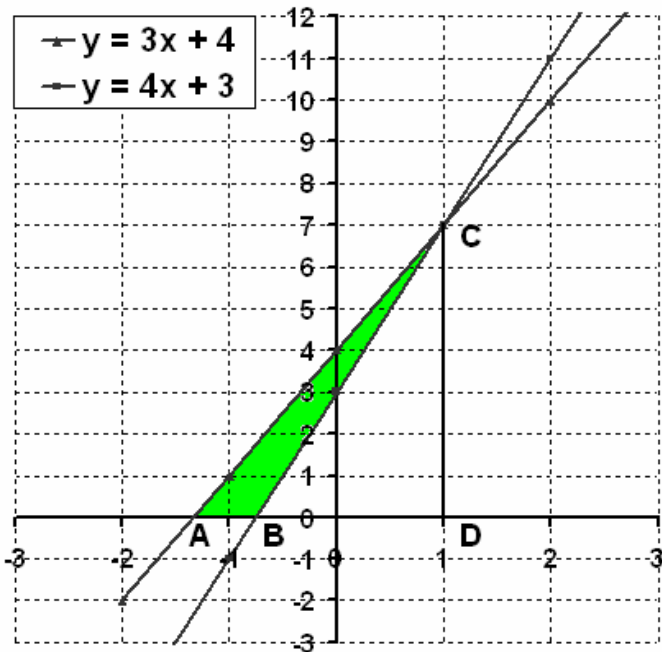
Yearly cost = 2,000,000 + 8,400,000 = 10,400,000.

Answer: The garden costs 10,400,000 soom per year

### Question 4

Draw the lines  $y = 3x + 4$  and  $y = 4x + 3$  showing clearly the point where they meet and work out the area of the triangle formed between these two lines and the x-axis.

**Solution:**



$$y = 3x + 4$$

$$x = 0 \Rightarrow y = 3 \cdot 0 + 4 = 4 \Rightarrow (0, 4)$$

$$y = 0 \Rightarrow 0 = 3x + 4 \Rightarrow x = -4/3 \Rightarrow A(-4/3, 0)$$

$$y = 4x + 3$$

$$x = 0 \Rightarrow y = 4 \cdot 0 + 3 = 3 \Rightarrow (0, 3)$$

$$y = 0 \Rightarrow 0 = 4x + 3 \Rightarrow x = -3/4 \Rightarrow B(-3/4, 0)$$

$$\begin{cases} y = 3x + 4 \\ y = 4x + 3 \end{cases} \Rightarrow 3x + 4 = 4x + 3 \Rightarrow$$

$$x = 1, y = 3 \cdot 1 + 4 = 7 \Rightarrow C(1, 7)$$

$$S_{ABC} = S_{ACD} - S_{BCD} = \frac{1}{2} AD \cdot CD - \frac{1}{2} BD \cdot CD =$$

$$= \frac{1}{2} \cdot \left(1 - \left(-\frac{4}{3}\right)\right) \cdot 7 - \frac{1}{2} \cdot \left(1 - \left(-\frac{3}{4}\right)\right) \cdot 7 =$$

$$= \frac{49}{6} - \frac{49}{8} = \frac{49}{12} = 4\frac{1}{12}$$

Answer:  $4\frac{1}{12}$ .

### Question 5

A person is employed at a salary of £18,000 (eighteen thousand pounds) per year, to be increased by £1,800 a year at the end of each year. He saves one quarter of his salary every year. Find the total saved at the end of 10 years.

**Solution:**

$$18,000, 19,800, \dots \quad a_1 = 18000, \quad d = 1800$$

$$a_{10} = a_1 + 9d = 18000 + 9 \cdot 1800 = 18000 + 16200 = 34200$$

$$S_{10} = \frac{a_1 + a_{10}}{2} \cdot 10 = \frac{18000 + 34200}{2} \cdot 10 = 52200 \cdot 5 = 261,000$$

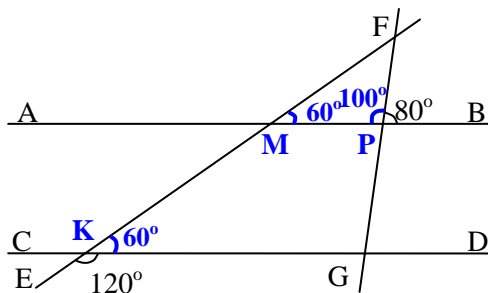
$$\frac{1}{4} S_{10} = \frac{1}{4} \cdot 261,000 = 65,250$$

Answer: 65,250.

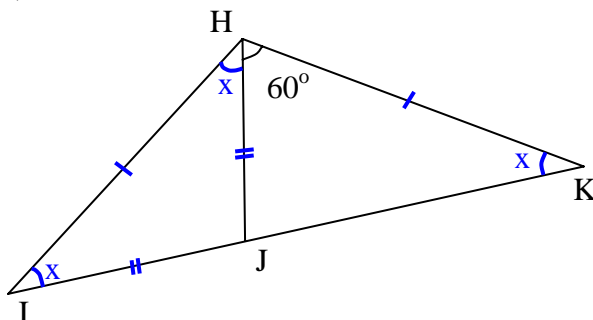
### Question 6

Find a) the angle  $\angle EFG$  if AB is parallel to CD, and b) the angle  $\angle IHK$  if HI = HK, HJ = JI and  $\angle JHK = 60^\circ$ .

a)



b)



**Solution:** a)  $\angle FMP = \angle FKG = 60^\circ$ ;  $\angle FPM = 180^\circ - \angle BPF = 180^\circ - 80^\circ = 100^\circ$ ;

For  $\triangle FMP$ :  $60^\circ + 100^\circ + \angle MFP = 180^\circ \Rightarrow \angle MFP = 20^\circ = \angle EFG$ .

b)  $HI = HK \Rightarrow \angle HIJ = \angle HKJ = x$ ;  $HJ = JI \Rightarrow \angle HIJ = \angle JHI = x$

For  $\triangle HIK$ :  $x + x + (x + 60^\circ) = 180^\circ \Rightarrow 3x = 120^\circ \Rightarrow x = 40^\circ \Rightarrow \angle IHK = 40^\circ + 60^\circ = 100^\circ$ .

Answer: a)  $\angle EFG = 20^\circ$  and b)  $\angle IHK = 100^\circ$