March 2009
WIUT exam paper solutions

Question 1

Solve $(x^2+\frac{x}{2}-10)^2-(x^2-\frac{x}{2}-8)^2=0$

Solution:

Method A:

If we regard $(x^2+\frac{x}{2}-10)$ as a, and $(x^2-\frac{x}{2}-8)$ as b, the expression can be simplified using the identity $a^2-b^2=(a+b)(a-b)$. Then we will have:

$(x^2+\frac{x}{2}-10)^2-(x^2-\frac{x}{2}-8)^2=[(x^2+\frac{x}{2}-10)+(x^2-\frac{x}{2}-8)]\,[(x^2+\frac{x}{2}-10)-(x^2-\frac{x}{2}-8)]=$

$=[2x^2-18]\,[x-2]=0$

The product becomes zero, only when one of the factors is 0, thus there may be two cases:

1) $x-2=0 \Rightarrow x_1=2$
2) $2x^2-18=0 \Rightarrow x^2=9 \Rightarrow x_2=-3,\quad x_3=3$

Method B:

$(x^2+\frac{x}{2}-10)^2-(x^2-\frac{x}{2}-8)^2=0 \quad\Rightarrow\quad (x^2+\frac{x}{2}-10)^2=(x^2-\frac{x}{2}-8)^2$

If $A^2=B^2$ then A must be equal to B or –B. That is, $A=\pm B$

We have $(x^2+\frac{x}{2}-10)^2=(x^2-\frac{x}{2}-8)^2$ and two cases should be examined:

1) A=B $\Rightarrow x^2+\frac{x}{2}-10=x^2-\frac{x}{2}-8 \Rightarrow \frac{x}{2}+\frac{x}{2}=-8+10 \Rightarrow x_1=2$
2) A=-B $\Rightarrow x^2+\frac{x}{2}-10=-(x^2-\frac{x}{2}-8) \Rightarrow x^2+x^2=8+10 \Rightarrow x^2=9 \Rightarrow x_2=-3,\quad x_3=3$

Answer: $x=-3$, $x=2$ or $x=3$

Question 2

Solve $\frac{x}{4}\left(3-\frac{8}{x}\right)-\frac{7}{8}\left(7-\frac{3x}{4}\right)=15\left(\frac{1}{3}-\frac{x}{64}\right)$.

Solution:

$$\frac{x}{4}\left(3-\frac{8}{x}\right)-\frac{7}{8}\left(7-\frac{3x}{4}\right)=15\left(\frac{1}{3}-\frac{x}{64}\right) \Rightarrow \frac{x}{4}\left(3-\frac{8}{x}\right)-\frac{7}{8}\left(7-\frac{3x}{4}\right)-15\left(\frac{1}{3}-\frac{x}{64}\right)=0$$

$$\frac{3x}{4}-\frac{8x}{4x}-\frac{49}{8}+\frac{21x}{32}-\frac{15}{3}+\frac{15x}{64}=0 \Rightarrow \frac{3x}{4}-2-\frac{49}{8}+\frac{21x}{32}-5+\frac{15x}{64}=0$$

$$\frac{3x*16}{4*16}+\frac{21x*2}{32*2}+\frac{15x}{64}=\frac{56}{8}+\frac{49}{8} \Rightarrow \frac{48x}{64}+\frac{42x}{64}+\frac{15x}{64}=\frac{56+49}{8}$$

$$\Rightarrow \frac{105x}{64}=\frac{105}{8} \Rightarrow \frac{x}{64}=\frac{1}{8} \Rightarrow x=\frac{64}{8}=8$$

Answer: $x=8$

Question 3

A group of friends paid £42 for 3 pizzas, 4 hamburgers and 1 cheeseburger. Another group paid £58 for 2 pizzas, 8 hamburgers, 1 salad and 2 cheeseburgers using the same price list. Fill in the price list below.

West Foods
Price List

Salad	£2
Hamburger	<u>£3.75</u>
Cheeseburger	£6
Pizza	<u>£7</u>

£ is a British 'pound'

Solution:

1) Let the price of hamburger be H, and that of pizza be P.
2) The first group paid 3*P for 3 pizzas, 4*H for 4 hamburgers and £6 for 1 cheeseburger and spent £42. That is, 3*P+4*H+£6=£42
3) The second group however bought 2 pizzas for 2*P, 8 hamburgers for 8*H, 1 salad for £2 and 2 cheeseburgers for £12. They spent £58 in total. From this information the following equation be derived: 2*P+8*H+£2+£12=£58
4) Solving these two equations we can find the values of P and H:

$$\begin{cases} 3P+4H+6=42 \\ 2P+8H+2+12=58 \end{cases} \Rightarrow \begin{cases} 3P+4H=36 \\ 2P+8H=44 \end{cases} \Rightarrow + \begin{cases} -6P-8H=-72 \\ 2P+8H=44 \end{cases} \quad \overline{-4P=-28} \Rightarrow P=7$$

⇨ 3*7+4H=36 => 4H=15 => H=3.75

So, the <u>price of pizza is £7</u> and the <u>price of hamburger is £3.75</u> (3 pounds and 75 pennies)

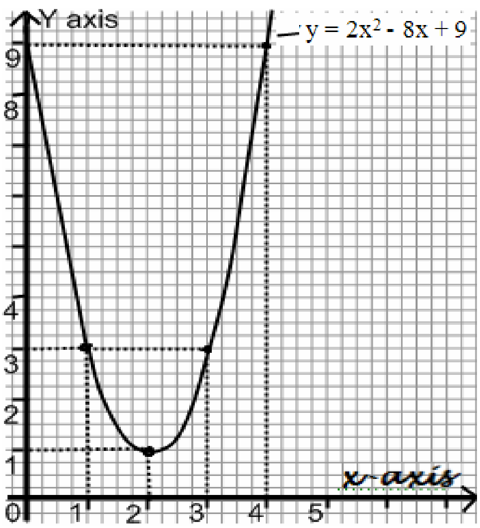
Question 4

Draw the curve represented by $y = 2x^2 - 8x + 9$ for $x \geq 0$, stating and showing clearly the point where the curve turns.

Solution:

The function is quadratic, thus the graph will be a parabola. As the coefficient of x^2 is positive (a=2), the branches of the parabola will be "looking upwards".

1) The graph intersects Y axis at $y(0)=2*0^2-8*0+9=9$. That is, at the point (0; 9);
2) The discriminant of $2x^2 - 8x + 9$ is $D=b^2-4ac= (-8)^2-4*2*9=-8$. As D<0, the graph does not intersect the x axis;
3) Now let`s find the turning point (vertex) of the parabola: The abscissa (X value) of the turning point can be found by the formula $x_0 = -\frac{b}{2a}$ =>
$x_0 = -\frac{-8}{2*2} = 2$ => $x_0= 2$. The ordinate (Y value) can be found by calculating the value of the function at x_0 => $y_0=y(2)=2*2^2-8*2+9= 1$. So the vertex of the parabola is at (2;1)
4) To draw the graph of the parabola accurately, we need to find some more of its points:
   $y(1)= 2*1^2-8*1+9=3$ => (1;3);
   $y(3)=2*3^2-8*3+9=3$ => (3;3);
   $y(4)=2*4^2-8*4+9=9$ => (4;9).



X	0	1	2	3	4
Y	9	3	1	3	9

5) The graph of the parabola can now be drawn using the above points.

Question 5

Anna added consecutive even numbers starting from 2 and reached a total of 62,750 (sixty two thousand seven hundred and fifty). How many numbers did she manage to add?

Solution:

Step 1) The consecutive even numbers make up an arithmetic progression, where first term is $a_1=2$ and the common difference is d=2.

Step 2) Anna added up starting from a_1, to a_n until the sum (S_n) became 62750. Here we can use the formula of the sum of n terms of the arithmetic progression: $S_n=\frac{a_1+a_n}{2}n$ As $a_n=a_1+(n-1)d$, the formula can be rewritten as

$S_n=\frac{a_1+a_1+(n-1)d}{2}n=\frac{2+2+2n-2}{2}n=(n+1)n=62750$. The value of n can be found solving the equation $(n+1)n=62750$ => $n^2+n-62750=0$ => D=1-4(-62750)=251001=>

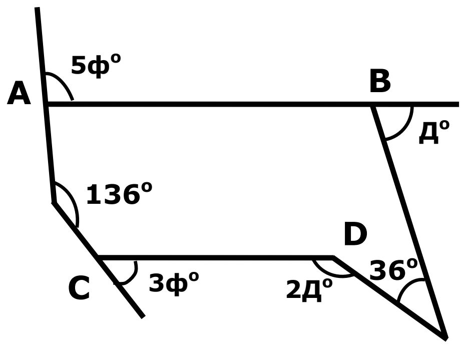
$n_1=\frac{-1-\sqrt{251001}}{2}=-251$ and $n_1=\frac{-1+\sqrt{251001}}{2}=250$

The root n_1=-251 is rejected as the number of terms can not be a negative number. n=250 can be accepted. So, she added 250 consecutive even numbers.

Answer: She added together 250 numbers

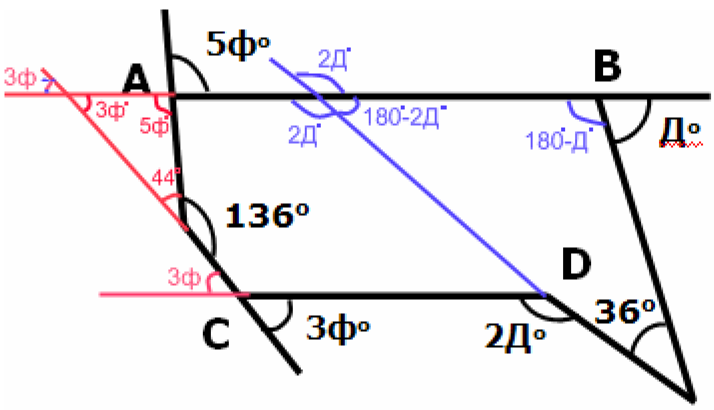
Question 6

AB is parallel to CD. Find the value of Д and ф.



Solution:

One of the methods of solution is to construct new triangles by stretching the lines:



1) The angles of the first triangle constructed (using red lines) are $3ф^0$, $5ф^0$ and 44^0. It is well-known that for any triangle, the sum of the internal angles is 180^0. Thus we derive an equation:
$3ф^0+5ф^0+44^0=180^0$ => $ф^0=17^0$

2) From the second triangle we get an equation:
$36^0+(180-2Д^0)+(180^0-Д^0)=180^0$
=> $36^0+180^0=3Д^0$ => $Д^0=72^0$

Answer: $ф^o=17^o$, $д^o=72^o$