February 2009
WIUT exam paper solutions

Question 1
Solve $(x^2 + 4x - 4)^2 - (x^2 - 4x - 4)^2 = 0.$

Solution.
This problem can be solved using several methods:

Method A:
We can simplify the expression using the identity $a^2 - b^2 = (a+b)(a-b)$. Then we will have $(x^2 + 4x - 4)^2 - (x^2 - 4x - 4)^2 = [(x^2 + 4x - 4) + (x^2 - 4x - 4)]\,[(x^2 + 4x - 4) - (x^2 - 4x - 4)] =$ $=[2x^2 - 8]\,[8x]=0$ As the product will be 0 when one of the factors is zero, we will examine two cases where

1) $8x=0$ $\Rightarrow x_1=0$
2) $2x^2 - 8=0$ $\Rightarrow x_2=-2, \quad x_3=2$

Method B:
$(x^2 + 4x - 4)^2 - (x^2 - 4x - 4)^2=0 \quad \Rightarrow \quad (x^2 + 4x - 4)^2 = (x^2 - 4x - 4)^2$
As we know, if $a^2 = b^2$ then $a=\pm b$
We have $(x^2 + 4x - 4)^2 = (x^2 - 4x - 4)^2$ and thus two cases should be examined:

1) $(x^2 + 4x - 4) = (x^2 - 4x - 4) \Rightarrow 4x= -4x \Rightarrow 8x = 0 \Rightarrow x_1=0$
2) $(x^2 + 4x - 4) = -(x^2 - 4x - 4) \Rightarrow 2x^2 - 8=0 \Rightarrow x_2=-2, \quad x_3=2$

Method C:
We can just open the brackets. This method is a bit confusing, thus requires more attention and accuracy:
$(x^2 + 4x - 4)^2 - (x^2 - 4x - 4)^2 = (x^2 + 4x - 4)(x^2 + 4x - 4) - (x^2 - 4x - 4)(x^2 - 4x - 4) =$
$= (x^4 + 4x^3 - 4x^2 + 4x^3 + 16x^2 - 16x - 4x^2 - 16x + 16) - (x^4 - 4x^3 - 4x^2 - 4x^3 + 16x^2 + 16x - 4x^2 + 16x + 16) =$
$16x^3 - 64x = 16x(x^2 - 4) = 0$ As the product will be 0 when one of the factors is zero, we will examine two cases where

1) $16x=0$ $\Rightarrow x_1=0$
2) $x^2 - 4=0$ $\Rightarrow x_2=-2, \quad x_3=2$

You can use any method and be awarded a full mark provided that you show the calculations and solve it correctly.

Answer: $x = -2$, $x = 0$ or $x = 2$

Question 2

Find the value of $A-B+3\left[\left\{A-B+AB\left(\frac{1}{A}+\frac{1}{B}\right)\right\}^2-4A^2+B\right]$ when $A=\frac{2}{3}, B=-\frac{1}{3}$.

Solution:

Before calculating its value for $A=\frac{2}{3}, B=-\frac{1}{3}$, it is better to simplify the expression:

Step 1: $AB\left(\frac{1}{A}+\frac{1}{B}\right)=AB\left(\frac{A+B}{AB}\right)=A+B$

Step 2: $\left\{A-B+AB\left(\frac{1}{A}+\frac{1}{B}\right)\right\}^2=\{A-B+(A+B)\}^2=(2A)^2=4A^2$

Step 3:

$$A-B+3\left[\left\{A-B+AB\left(\frac{1}{A}+\frac{1}{B}\right)\right\}^2-4A^2+B\right]=A-B+3\left[4A^2-4A^2+B\right]=A-B+3B=A+2B$$

So, the expression has become A+2B after simplification. Now, it is very easy to find its value for $A=\frac{2}{3}, B=-\frac{1}{3}$ which is $A+2B=\frac{2}{3}+2\left(\frac{-1}{3}\right)=\frac{2}{3}-\frac{2}{3}=0$

Answer: The value is = 0

Question 3

An egg dealer bought a number of eggs at 60p for 6, and five times that number for 900p for 100. He sold them all at 72p per 6 eggs and made a profit of 1020p. How many eggs did he buy?

p is a 'penny' [British money]

Solution:

When solving such kind of problems, it is very important to rewrite its mathematical expression.

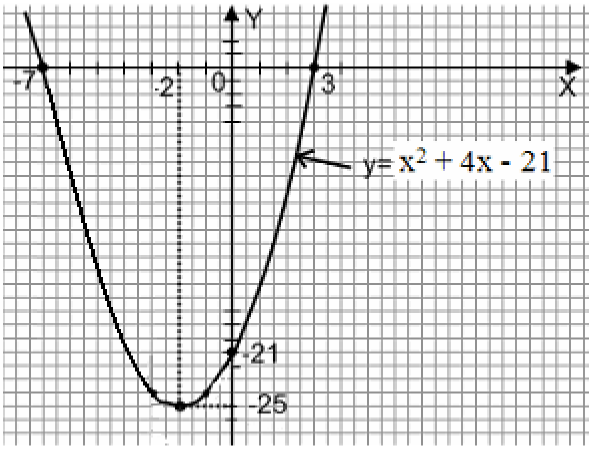
1) Here, we are not given the number of eggs. Let the number of eggs bought first time be X (You can use any other letter if you want). Then the number of eggs bought second time will be 5X.
2) In his first purchase, the dealer bought the eggs at a price of 60pennies/6 eggs => 10 pennies per an egg. Second time the price was 900 pennies/100 eggs => 9 pennies per an egg.
3) Now, we can calculate how much money he spent to buy the eggs: X eggs at a price of 10 pennies per egg + 5X eggs at a price of 9 pennies per an egg. That is, 10X +9*5X=55X pennies.
4) Now we calculate his income: he sold X+5X=6X eggs at a price of 72 pennies per 6 eggs, that is 12 pennies per an egg. His income 12*6X=72X pennies
5) The profit = Income-Expenditure is given to be 1020. => 72X pennies -55X pennies= 17X pennies=1020 pennies => 17X=1020 => X=60 eggs

So, the dealer bought 60 eggs at the first purchase. In total he bought 60+5*60=360 eggs.

Answer: He bought 360 eggs

Question 4
Draw the curve represented by $y = x^2 + 4x - 21$ stating and showing clearly the point where the curve turns and the points where it meets the axes.

Solution:



The function is quadratic, it means the graph will be a parabola. The coefficient of x^2 is positive (a=1), it means the branches of parabola will be "looking upwards".
We find where the graph intersects the X and Y axes:

1) The graph intersects Y axis when x=0 => $y=0^2+4*0-21=-21$.

That is, at the point (0; –21)

2) The graph intersects X axis when y=0 => $x^2+4x-21=0$ => $x_1=-7$ and $x_2=3$

That is, at the points (–7;0) and (3;0)

3) The question is also asking to show the turning point of the parabola. The abscissa (X value) of the turning point can be found by solving the equation $2ax_0 +b=0$. => $2X_0+4=0$ => $X_0=-2$. The ordinate (Y value) can be found by calculating the value of the function at X_0. => $Y_0=(-2)^2+4(-2)-21=-25$
4) The graph of the parabola can now be drawn using these 4 points.

Question 5
The 3rd and the 20th term of an Arithmetic Progression are 7 and 58. Find the 50th term.

Solution:
Step 1) We are given $a_3=7$ and $a_{20}=58$. The common difference of the progression can be found by subtracting any two terms of the arithmetic progression:
$a_{20}- a_3= 58-7=51$ => $a_{20}- a_3 = (a_1+19d) - (a_1+2d) = 17d=51$ => d=3

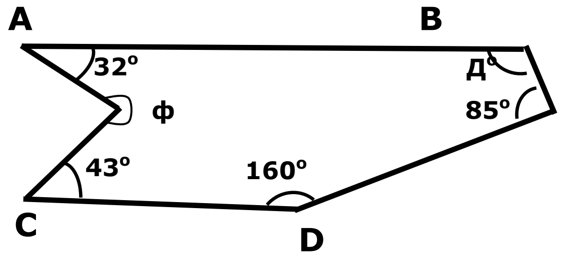
Step 2) As $a_3=a_1+2d$ => $a_1=a_3-2d=7-2*3=1$

Step 3) $a_n=a_1+(n-1)d$ => $a_{50}=a_1+49d= 1 + 49*3=148$

Answer: The 50th term is = 148

Question 6

Line AB is parallel to line CD. Find the value of ф and д.

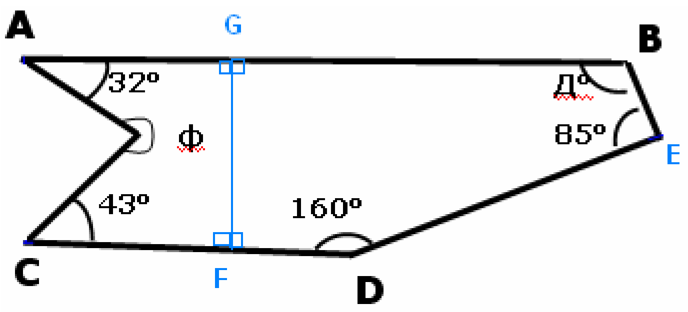


Solution:

There are several methods to solve this problem. Below only one of the methods is shown:

Method A:

A



A line FG should be drawn perpendicular to AB and CD. Now, we have two pentagons: AGFCф and GBEDF. As the sum of the internal angles of any pentagon is 540° (from the formula (n-2)180°), the value of the angles д and ф can be found from equations:

90° +90° +32° +43° + ф° = 540°

=> ф° = 285°

90° +90° +160° +85° + д° =540°

=> д° = 115°

The problem can also be solved by stretching the line from C to the side AB where the angle will also be 43°, stretching the line from A to the side CD and so on.

Answer: ф° = 285°, д° = 115°